Accurate h-Conform Finite Element Model of Multiply Connected Thin Regions via a Subproblem Method

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***Abstract*—A subproblem method for solving eddy current finite element is developed to correct the inaccuracies near edges and corners arising from thin shell models. Such models replace thin volume regions by surfaces but neglect border effects in the vicinity of their edges and corners. A thin shell solution, performed by a simplified mesh near the thin structures, serves as a source of a correction problem consisting of the actual volume thin regions alone and concentrating the meshing effort on the thin regions. The general case of multiply connected thin regions is considered.**

I. Introduction

Thin shell (TS) finite element (FE) representation of thin regions in magnetic problem is herein placed at the hearth of the subproblem method (SPM) [3], to define both parameterized and correction schemes. The TS models are commonly used to avoid meshing the thin rgions, and consequently lighten the mesh of their surrounds [1]. Indeed, the volume thin regions are reduced to a zero-thickness double layer with interface conditions (ICs) linked to 1-D analytical distributions along the shell thickness that however generally negect end and curvature effects.

The SPM for the $\boldsymbol{h}$-conform FE formulation has been already developed by authors [3] for simply connected TS regions, proposing a surface-to-volume local correction. The method is herein extended to multiply connected TS regions, i.e. regions with holes, for both the associated surface model and its volume correction. The global currents flowing around the holes and their associated voltages are naturally coupled to the local quatities, via some cuts for magnetic scalar potential discontinuities (Fig. 1) at both TS and correction steps.

II. Thin Shell Solution and its correction

The magnetic field $\boldsymbol{h}_i$ of the SPM is split into a reation field $\boldsymbol{h}_{r,i}$ and a source field $\boldsymbol{h}_{s,i}$ due to the imposed current density $\boldsymbol{j}_{s,i}$. In non-conducting regions $\Omega_{c,i}^C$, $\boldsymbol{h}_{r,i}$ is defined via a magnetic scalar potential ϕ_i such that $\boldsymbol{h}_{r,i} = -\text{grad}\,\phi_i$ [1] [2]. Potential ϕ_i in $\Omega_{c,i}^C$ can be multivalued (Fig. 1) and is then made singlevalued via the definition of cuts through each hole of $\Omega_{c,i}$ [2].

A first problem involving only stranded inductors is solved on a simplified mesh without thin regions. Its solutions gives SSs for a TS problem via ICs. The TS solution is then corrected by a correction solution that overcomes the TS assumptions [1]. The SPM approach proposed in [3] provides the tools to perform such a model refinement, thanks to simultaneous surface sources (SSs) and volume sources (VSs). SSs related to ICs [1], [3] compensate the TS and cut discontinuities, in parallel to VSs in the added volume shell that account for volume change of the permeability and conductivity [3]. Non-conducting or conducting regions can be considered on both sides of the thin regions.

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III. Application Example

The 3D test problem is the TEAM problem 7 (coil and thin plate with a hole, Fig. 2) ($\mu_r = 1$, σ = 35.26 MS/m). A SP scheme considers the TS model followed by its volume correction. Distributions of eddy current densities on TS SP and volume correction SP are shown in Fig. 2. The error on TS SP locally reaches 52%. The inacuracy on the Joule power loss densities of TS problem is pointed out by the importance of the correction SP and reaches 53% along the plate borders, with thickness d = 19 mm and frequency f = 200 Hz (skin depth δ = 4.89 mm). For d = 19 mm and f = 200 Hz, the TS error is 13% for the global current and 42% for the Joule loss (reduced to 26% for f = 50 Hz). For d = 2 mm and f = 200 Hz, it is respectively reduced to 1% and 6% (4% for f = 50 Hz).

The SPM allows to accurately correct any TS solution. In particular, accurate correction of eddy current and power loss density are obtained at the edges and corners of multiply connected thin regions. Details on the proposed method will be given in the extended paper.

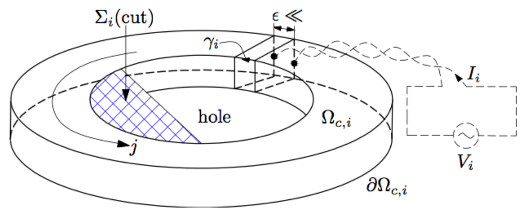

Fig. 1. Typical multiply connected 3D eddy-current problem.

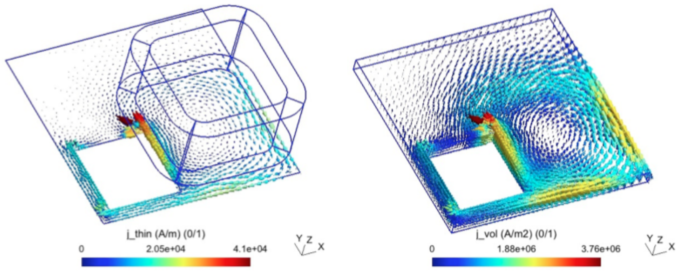

Fig. 2. Eddy current density on TS model (*left*) and volume correction (*right*) (d = 19 mm, f = 200 Hz, $\mu_{r,plate} = 1$, σ_{plate} = 35.26 MS/m).

References

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